

## Transfer of polarized infrared radiation in optically anisotropic media: application to horizontally oriented ice crystals: comment

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The calculations of multiple scattering of polarized infrared radiation by horizontally oriented ice crystals in a paper by Takano and Liou [J. Opt. Soc. Am. A **10**, 1243 (1993)] are based on a vector radiative transfer equation that is not rigorous but rather should be considered approximate. Specifically, Takano and Liou use the scalar extinction coefficient instead of the  $4 \times 4$  extinction matrix. Therefore additional computations based on the rigorous radiative transfer equation are necessary for ascertaining the accuracy of Takano and Liou's radiative transfer calculations. Also, contrary to the statement of Takano and Liou, the reciprocity relation for the phase matrix holds for any anisotropic scattering media, including horizontally oriented ice crystals.

### 1. RADIATIVE TRANSFER EQUATION

In a recent paper<sup>1</sup> Takano and Liou calculate the transfer of polarized infrared radiation in an optically anisotropic medium composed of horizontally oriented ice crystals. Unfortunately, they do not point out that their consideration of multiple scattering of polarized light is based on a vector radiative transfer equation that is not rigorous but rather should be considered approximate. Indeed, in their Eq. (1), as well as in Eq. (2) of Ref. 2 and Eq. (5.5.2) of Ref. 3, which they call the general radiative transfer equation for horizontally oriented crystals, they use the scalar extinction coefficient  $\beta_e$ , whereas for optically anisotropic media this scalar extinction coefficient must be replaced by the  $4 \times 4$  extinction matrix  $\Lambda$ .<sup>4</sup> For isotropic media composed of randomly oriented particles having a plane of symmetry and (or) particles and their mirror particles in equal numbers with random orientation, the use of the extinction coefficient is justified, because in this case the extinction matrix is diagonal and is equal to  $\beta_e \mathbf{E} = \rho C_{\text{ext}} \mathbf{E}$ , where  $\rho$  is the number density of scattering particles,  $C_{\text{ext}}$  is their (averaged) extinction cross section, and  $\mathbf{E} = \text{diag}\{1, 1, 1, 1\}$  is the  $4 \times 4$  unity matrix. However, for anisotropic media composed of perfectly or partially oriented nonspherical scatterers, some or all of the off-diagonal elements of the extinction matrix are nonzero. This phenomenon is called linear dichroism of the scattering medium<sup>5,6</sup> and results in different values of extinction for differently polarized light. Two well-known manifestations of this phenomenon are interstellar polarization<sup>7,8</sup> (polarization of transmitted light by nonspherical interstellar dust grains partially aligned in cosmic magnetic fields) and cross polarization of radio waves coherently propagating through partially aligned nonspherical hydrometeors.<sup>9,10</sup>

An important consequence of linear dichroism is that, in general, the optical depth (thickness) and the albedo for single scattering cannot be introduced at all.<sup>11</sup> Correspondingly, the main equations of the radiative transfer theory, such as the adding/doubling and invariant embedding equations, are substantially modified, as discussed in detail in Ref. 11.

In general, all 16 elements of the extinction matrix may be nonzero and are functions of the direction of light propagation  $\mathbf{n}$ . In the case of nonspherical particles randomly oriented in the horizontal plane (or, more generally, axially oriented<sup>8</sup>), the extinction matrix becomes simpler and has the form

$$\Lambda(\mathbf{n}) \equiv \Lambda(\mu) = \rho \begin{bmatrix} C_{\text{ext}}(\mu) & C_{\text{pol}}(\mu) & 0 & 0 \\ C_{\text{pol}}(\mu) & C_{\text{ext}}(\mu) & 0 & 0 \\ 0 & 0 & C_{\text{ext}}(\mu) & C_{\text{cpol}}(\mu) \\ 0 & 0 & -C_{\text{cpol}}(\mu) & C_{\text{ext}}(\mu) \end{bmatrix}, \quad (1)$$

where  $C_{\text{ext}}$ ,  $C_{\text{pol}}$ , and  $C_{\text{cpol}}$  are cross sections for extinction, linear, and circular polarization, respectively;  $\mu$  is the cosine of the zenith angle; and the standard  $(I, Q, U, V)$  representation of polarization<sup>5</sup> is assumed. Clearly, in computations of multiply scattered infrared radiation, as discussed by Takano and Liou,<sup>1</sup> only the upper-left  $2 \times 2$  submatrix of this matrix is important. It also follows from Eq. (1) that in the computations of Takano and Liou<sup>1</sup> the cross section for linear polarization was implicitly neglected. Unfortunately, it is difficult to estimate *a priori* the magnitude of the error in the multiple-scattering computations of Takano and Liou<sup>1</sup> introduced by the neglect of  $C_{\text{pol}}$ . Therefore it would be

important to compare their data with numerical solutions of the rigorous radiative transfer equation. Note that in Ref. 12 the eigenvalue–eigenvector technique was used to solve this rigorous radiative transfer equation for an azimuthally symmetric anisotropic medium composed of perfectly aligned prolate and oblate spheroids. In Ref. 13, multiple scattering of polarized light for a layer of axially oriented small spheroids is calculated on the basis of first- and second-order vector radiative transfer theory, and the effects of the nondiagonal extinction matrix are shown to be significant.

## 2. SYMMETRY RELATIONS

On pages 1249 and 1250 of Ref. 1, Takano and Liou claim that Hovenier's<sup>14</sup> symmetry relation (B) for the phase matrix, which has the following general form,

$$\mathbf{Z}(-\mathbf{n}', -\mathbf{n}) = \mathbf{P} \mathbf{Z}^T(\mathbf{n}, \mathbf{n}') \mathbf{P}, \quad (2)$$

and is valid for isotropic scattering media, is not applicable to the case of horizontally oriented ice crystals. In Eq. (2),  $\mathbf{P} = \text{diag}\{1, 1, -1, 1\}$ , and  $T$  denotes the transposed matrix. However, as was shown in Display 3.1 of Ref. 15 and in Ref. 11, this symmetry relation is a direct consequence of Saxon's<sup>16</sup> reciprocity relation for the amplitude scattering matrix and is valid for any anisotropic media, including horizontally oriented ice crystals. A similar general reciprocity relation holds for the extinction matrix<sup>11</sup>:

$$\Lambda(-\mathbf{n}) = \mathbf{P} \Lambda^T(\mathbf{n}) \mathbf{P}. \quad (3)$$

An important consequence of Eqs. (2) and (3) is general reciprocity relations for the reflection and transmission matrices of a plane-parallel anisotropic medium given by Display 3.2 of Ref. 15 and by Eqs. (51)–(53) of Ref. 11. Those reciprocity relations may be used in computer calculations to reduce CPU time and storage requirements substantially.

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